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Quantum Criticality in Silicene

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We study possibility of superconducting state in silicene. We see the possibility of spin singlet superconductivity, s wave and p+ip wave, close to dirac point. p+ip wave state is possible due to special structure of honey comb lattice because it has two inequivalent fermi surfaces at dirac points. We find superconducting gaps and find that gap for p+ip phase is proportional to spin orbit coupling strength. Quantum critical point exist for both S wave and p+ip wave. Superconductivity in slicene for S wave exist in the weak coupling regime and for p+ip wave in the strong coupling regime. We also find zero temperature phase transition quantum critical point for s wave states.

Silicene is a low bulked sheet of atoms of silicone forming honey comb lattice[11][12]. Silicene got much attraction due to existence of dirac fermions[13][14]. In silicene quantum Hall effect is proved due to strong spin orbit coupling[15][16]. The 1 and 2 dimension s and $p+ip$ superconductors with Majorana fermions are studied in Ref[17]. Which also deal with bulk properties of 2 dimensional bulk superconductors. The superconductivity due to electron-phonon interaction in silicene under biaxial strain with in the rigid band approximiation is studied in Ref[18]. Experimentally possible superconducting gap has been studied by Lan Chen, Baojie Feng and kehui Wu[19] in silicene epitaxially grown on Ag(111) surface. S wave superconducting gap in presence of inhomogeneous electric field is discussed by Motohiko Ezawa Yukio Tanaka and Naoto Nagaosa in Ref[20].

In present work we derive mean field superconducting s wave and $p+ip$ gap for silicene. we see that in the strong coupling regime there is quantum critical point. In silicene A and B sub lattice donot lie in same plane and silicene has bulked structure due sp^3 hybridization with distance $2l$ between two planes[2], where $l = .23A^0$. In silicene valance electron are in 3rd shell. Due to increase in shell the spin orbit coupling is dominant as compare to graphene.
Hamiltonian for silicene is written as[2].

$$H_{silicene} = -t\sum_{\langle i,j\rangle}\sum_{s}(a^{\dagger}_{is}b_{js} + b^{\dagger}_{js}a_{is})$$
$$+ i\frac{\lambda_{so}}{3\sqrt{3}}\sum_{\langle\langle ij\rangle\rangle}\sum_{s}S^{s}_{z}v_{ij}(a^{\dagger}_{is}a_{js} + b^{\dagger}_{is}b_{js})$$

First term is nearest neighbor hopping term and second is the effective spin orbit coupling. Where t is the hopping energy, λ_{so} is spin orbit coupling strength, $a^{\dagger}$ (a) is the creation (annihilation) operator for sublattice A and $b^{\dagger}$ (b) is the creation (annihilation) operator for sublattice B. Where $v_{ij} = \frac{\delta_i\times\delta_j}{|\delta_i\times\delta_j|}$ [10], where δ_i and δ_j are two nearest bonds connecting next nearest neighbors Υ, S^s_z is the z component of Pauli spin matrix and ss' are the spin indices with $s =\uparrow,\downarrow$. $\delta_1, \delta_2, \delta_3$ are the vectors connecting nearest neighbors. $\delta_1 = \frac{a}{\sqrt{3}}(1, 0, \cot\theta)$, $\delta_2 = \frac{a}{\sqrt{3}}(\frac{-1}{2}, \frac{\sqrt{3}}{2}, \cot\theta)$, $\delta_3 = \frac{a}{\sqrt{3}}(\frac{-1}{2}, \frac{-\sqrt{3}}{2}, \cot\theta)$. Lattice vector for silicene are $\mathbf{a}_1 = \delta_1 - \delta_2$, $\mathbf{a}_1 = \frac{a}{2}(\sqrt{3}, 1)$, $\mathbf{a}_2 = \delta_1 - \delta_3$, $\mathbf{a}_2 = \frac{a}{2}(\sqrt{3}, -1)$ with $a = 3.86A^0$. The reciprocal lattice vector using the equation. $\mathbf{a}_i.\mathbf{b}_j = 2\pi\delta_{ij}$, is given by $\mathbf{b}_1 = \frac{2\pi}{a}(\frac{1}{\sqrt{3}}, 1)$, $\mathbf{b}_2 = \frac{2\pi}{a}(\frac{1}{\sqrt{3}}, -1)$. The first brillion zone has six corner with two distinct corner. All other corner can be written in the form of these two corner called two valley points k_1, k'_1.

$$\mathbf{K}_1 = \frac{2\pi}{\sqrt{3}a}(1, \frac{1}{\sqrt{3}})$$
$$\mathbf{K}'_1 = \frac{2\pi}{\sqrt{3}a}(1, \frac{-1}{\sqrt{3}})$$

The Fourier transform for annihilation operator is define as $a_{is} = \sum_k \exp^{ik.r_i} a_{ks}$, $b_{is} = \sum_k \exp^{ik.r_i} b_{ks}$ and $\sum_{\langle ij\rangle} \longrightarrow \sum_i\sum_{i+\delta}$ Where $\delta = \delta_1, \delta_2, \delta_3$. Hamiltonian in K space with $u_k = \sum_\delta \exp^{ik.\delta}$, $v_k = \sum_\Upsilon v_{ij}\exp^{ik.\Upsilon}$ where Υ are vectors connecting second nearest neighbors is given as

$$H_{silicene} = -t\sum_k\sum_s u_k a^{\dagger}_{ks}b_{ks} - t\sum_k\sum_s u^*_k b^{\dagger}_{ks}a_{ks}$$
$$+ i\frac{\lambda_{so}}{3\sqrt{3}}\sum_k\sum_s S^s_z(v_k a^{\dagger}_{ks}a_{ks} + v^*_k b^{\dagger}_{ks}b_{ks})$$
$$H_{Silicene} = \begin{pmatrix} a^{\dagger}_k & b^{\dagger}_k\end{pmatrix}\begin{pmatrix} v_k S_z & -tu_k \\ -tu^*_k & v^*_k S_z\end{pmatrix}\begin{pmatrix} a_k \\ b_k\end{pmatrix}$$
$$H = \psi^{\dagger}A\psi$$

After diagonalization it give dispersion relation.

$$\omega_k = \frac{1}{2}(-i\frac{\lambda_{so}S_z}{3\sqrt{3}}(v_k + v^{\star}_k) \pm \sqrt{-\frac{\lambda^2_{so}}{27}(v_k + v^{\star}_k)^2 + 4t^2|u_k|^2 + \frac{4}{27}|v_k|^2\lambda^2_{so}})$$

Near the dirac point to find the low energy effective hamiltonian we expand u_k and v_k near the dirac point. $u_{k_1+k} = u_{k_1} + ik.\sum_\delta \delta\exp^{ik_1.\delta}$, $u_{k_1} = 0$, $u_{k_1+k} = ik.\sum_\delta \delta\exp^{ik_1.\delta}$, $u_{k_1+k} = \frac{\sqrt{3}a}{4}[-(\sqrt{3}k_x + k_y) + i(\sqrt{3}k_y - k_x)]$, $|u_{k_1+k}| = \frac{\sqrt{3}a}{2}|k|$, $v_{k_1+k} = v_{k_1} + iv_{ij}k.\sum_\Upsilon \Upsilon\exp^{ik_1.\Upsilon}$, $v_{k_1} =$

$3\sqrt{3}i$, $ikv_{ij}.\sum_{\Upsilon}\Upsilon\exp^{ik_1.\Upsilon}=0$, $v_{k_1+k}=3\sqrt{3}i$. Using above results. Close to Dirac point dispersion relation takes form

$$\omega_k=\pm\sqrt{v_0^2|k|^2+\lambda_{so}^2} \tag{1}$$

where $v_0=\frac{\sqrt{3}at}{2}=5.52\times10^5\frac{m}{s}$ and $\lambda_{so}=3.9\times10^{-3}eV$

The dispersion relation in Eq(1) is dispersion relation got from low energy effective Hamiltonian. There is band gap at k=0 which is proportional to spin orbit coupling strength[4] [9]. We find the possibility of superconductivity from the coulomb interaction by using mean field approximation[5]. Full Hamiltonian for silicene is written as[2]

$$\begin{aligned}H_{silicene}&=-t\sum_{\langle i,j\rangle}\sum_s(a_{is}^\dagger b_{js}+b_{js}^\dagger a_{is})\\&+i\frac{\lambda_{so}}{3\sqrt{3}}\sum_{\langle\langle ij\rangle\rangle}\sum_s S_z^s V_{ij}(a_{is}^\dagger a_{js}+b_{is}^\dagger b_{js})\\&-i\frac{2}{3}\lambda_R\sum_{\langle\langle ij\rangle\rangle ss'}\mu_{ij}a_{is}^\dagger(\sigma\times d_{ij})_{ss'}^z a_{js'}\\&-i\frac{2}{3}\lambda_R\sum_{\langle\langle ij\rangle\rangle ss'}\mu_{ij}b_{is}^\dagger(\sigma\times d_{ij})_{ss'}^z b_{js'}\end{aligned}$$

Third and forth term are Rashaba terms. Where $d_{ij}=\frac{\Upsilon}{\Upsilon}$ are the unit vectors in direction of second nearest neighbors and $\mu_{ij}=\pm1$ for sub-lattice A(B). Close to the Dirac point

$$\begin{aligned}(Y_{(k)})_{ss'}&=\sum_{\Upsilon}\mu_{ij}(\sigma\times d_{ij})_{ss'}^z\exp^{ik.\Upsilon}\\&=3ia(k_y\sigma_x-k_x\sigma_y)\end{aligned}$$

Now complete Hamiltonian in k space is written as

$$\begin{aligned}H_{silicene}&=-t\sum_k\sum_s(u_k a_{ks}^\dagger b_{ks}+H.c.)\\&+iv_k\frac{\lambda_{so}}{3\sqrt{3}}\sum_k\sum_s S_z^s a_{ks}^\dagger a_{ks}+iv_k^\star\frac{\lambda_{so}}{3\sqrt{3}}\sum_k\sum_s S_z^s b_{ks}^\dagger b_{ks}\\&-i\frac{2}{3}\lambda_R\sum_{kss'}(Y_{(k)})_{ss'}a_{ks}^\dagger a_{ks'}-i\frac{2}{3}\lambda_R\sum_{kss'}(Y_{(k)}^\star)_{ss'}b_{ks}^\dagger b_{ks'}\\&+g_0\sum_k(\Delta_0(a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger+b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger)+H.c.)\\&+g_1\sum_k(u_k^\star\Delta_1(a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger-a_{-k\downarrow}^\dagger b_{k\uparrow}^\dagger)+H.c.)+E_0\end{aligned}$$

The last terms containing g_0, g_1 comes from electron electron interaction as in graphene. Where $E_0=-g_0|\Delta_0|^2-3g_1|\Delta_1|^2$ with Δ_0 and Δ_1 are the S wave and $p+ip$ order parameter respectively. The diagram for $p+ip$ order parameter in k space near the Dirac point and away from the Dirac point, are similar as for graphene[3].

$$\begin{aligned}H_{silicene}&=-t\sum_k u_k(a_{k\uparrow}^\dagger b_{k\uparrow}+b_{-k\downarrow}^\dagger a_{-k\downarrow})+H.c.\\&+i\frac{\lambda_{so}}{3\sqrt{3}}\sum_k v_k(a_{k\uparrow}^\dagger a_{k\uparrow}-b_{-k\downarrow}^\dagger b_{-k\downarrow})\\&+i\frac{\lambda_{so}}{3\sqrt{3}}\sum_k v_k^\star(-a_{-k\downarrow}^\dagger a_{-k\downarrow}+b_{k\uparrow}^\dagger b_{k\uparrow})\\&-i\frac{2}{3}\lambda_R\sum_k Y_{(k)}a_{k\uparrow}^\dagger a_{-k\downarrow}+i\frac{2}{3}\lambda_R\sum_k Y_{(k)}a_{-k\downarrow}^\dagger a_{k\uparrow}\\&+i\frac{2}{3}\lambda_R\sum_k Y_{(k)}^\star b_{-k\downarrow}^\dagger b_{k\uparrow}-i\frac{2}{3}\lambda_R\sum_k Y_{(k)}^\star b_{k\uparrow}^\dagger b_{-k\downarrow}\\&+g_0\sum_k(\Delta_0(a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger+b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger)+H.c.)\\&+g_1\sum_k(u_k^\star\Delta_1(a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger-a_{-k\downarrow}^\dagger b_{k\uparrow}^\dagger)+H.c.)+E_0\end{aligned}$$

$$H=\psi^\dagger A\psi+E_0$$

We ignore Rashba term and take $\Delta_0=0$.

$$A=\begin{pmatrix}iv_k\frac{\lambda_{so}}{3\sqrt{3}}&-tu_k&0&u_kg_1\Delta_1\\-tu_k^\star&iv_k^\star\frac{\lambda_{so}}{3\sqrt{3}}&u_k^\star g_1\Delta_1&0\\0&u_kg_1\Delta_1&iv_k^\star\frac{\lambda_{so}}{3\sqrt{3}}&tu_k\\u_k^\star g_1\Delta_1&0&tu_k^\star&iv_k\frac{\lambda_{so}}{3\sqrt{3}}\end{pmatrix} \tag{2}$$

with basis function

$$\psi^\dagger=\begin{pmatrix}a_{k\uparrow}^\dagger&b_{k\uparrow}^\dagger&a_{-k\downarrow}&b_{-k\downarrow}\end{pmatrix}$$

After diagonalization it give dispersion relation.

$$\omega_k=\frac{\frac{i\lambda_{so}}{3\sqrt{3}}(v_k+v_k^\star)-s\sqrt{4t^2|u_k|^2+(i\frac{\lambda_{so}}{3\sqrt{3}}(v_{k-}v_k^\star)-2\beta|u_k|g_1\Delta_1)^2}}{2}$$

$$s,\beta=\pm1$$

Close to dirac point it takes form.

$$\omega_{k,s,\beta}=\beta\sqrt{v_0^2|k|^2+(\lambda_{so}+s\frac{v_0|k|g_1\Delta_1}{t})^2}$$

For s wave spin singlet state assume $\Delta_1,\lambda_R=0$

$$A=\begin{pmatrix}iv_k\frac{\lambda_{so}}{3\sqrt{3}}&-tu_k&g_0\Delta_0&0\\-tu_k^\star&iv_k^\star\frac{\lambda_{so}}{3\sqrt{3}}&0&g_0\Delta_0\\g_0\Delta_0&0&iv_k^\star\frac{\lambda_{so}}{3\sqrt{3}}&tu_k\\0&g_0\Delta_0&tu_k^\star&iv_k\frac{\lambda_{so}}{3\sqrt{3}}\end{pmatrix}$$

After diagonalization it give dispersion relation.

$$\omega_k=\frac{i\frac{\lambda_{so}S_z}{3\sqrt{3}}(v_k+v_k^\star)\pm\sqrt{-\frac{\lambda_{so}^2}{27}(v_k+v_k^\star)^2+4t^2|u_k|^2+4g_0^2\Delta_0^2}}{2}$$

Close to dirac point s wave and $p+ip$ spectrum takes form respectively.

$$\omega_{k,s} = \pm\sqrt{v_0^2|k|^2 + g_0^2\Delta_0^2 + \lambda_{so}^2}$$

$$\omega_{k,s,\beta} = \beta\sqrt{[1+\frac{g_1^2\Delta_1^2}{t^2}](v_0k + \frac{s\frac{g_1\Delta_1}{t}}{1+\frac{g_1^2\Delta_1^2}{t^2}})^2 + (\frac{\lambda_{so}}{\sqrt{1+\frac{g_1^2\Delta_1^2}{t^2}}})^2}$$

Where $\beta = \pm 1$. s wave superconducting gap is $E_g = 2\sqrt{g_0^2\Delta_0^2+\lambda_{so}^2}$. The $p+ip$ wave superconducting gap is $E_g = 2\lambda_{so}/\sqrt{1+\frac{g_1^2\Delta_1^2}{t^2}}$. Which is re-scaled by factor $\frac{1}{\sqrt{1+\frac{g_1^2\Delta_1^2}{t^2}}}$. $p+ip$ wave gap is linearly scale with spin orbit coupling. when $\lambda_{so}=0$ then gap also vanishes. We obtain the self consist equation for s wave and p wave order parameter from free energy[6]. We get free energy from dispersion relation.

$$F = -k_BT\ln(Z)$$

For fermionic system partition function is given as

$$Z = \prod_k (1+\exp^{\frac{-\omega_{k,s\beta}}{k_BT}})$$

Quasi particle are interacting fermion.

$$F = -k_BT\sum_{k,\beta,s} ln(1+\exp^{-\frac{\omega_{k,\beta,s}}{k_BT}}) + E_0$$

Applying summation of β, we get

$$F = -k_BT2(\sum_{k,s}\ln[2\cosh\frac{\omega_{k,s}}{2k_BT}] - 2g_0\Delta_0^2 - 3g_1\Delta_1^2)$$

We find Δ_0 and Δ_1 by minimization of free energy.
For p wave

$$\Delta_1 = -\sum_{k,s}\frac{v_0k(\frac{v_0kg_1\Delta_1}{t}+s\lambda_{so})\tanh\frac{\omega_{k,s}}{2k_BT}}{6t\omega_{k,s}} \quad (3)$$

Self consistent equation for s wave is given as

$$1 = -\sum_{k,s}\frac{g_0\tanh\frac{\omega_{k,s}}{2k_BT}}{4\omega_{k,s}}$$

At T=0

$$1 = -\sum_{k,s}\frac{g_0}{4\omega_{k,s}}$$

Applying summation of s and converting sum to integration.

$$1 = -g_0\int_0^\kappa \frac{2\pi kdk}{8\pi^2\sqrt{v_0^2k^2+g_0^2\Delta_0^2+\lambda_{so}^2}} \quad (4)$$

Where κ is cutoff momentum.which is associated with linear dispersion relation close to the dirac points.
Now we introduce new cutoff momenta Λ. Which define surface of constant energy on dirac cone.

$$v_0\kappa = \sqrt{v_0^2\Lambda^2+g_0^2\Delta_0^2+\lambda_{so}^2}$$

This approximation is used because results do not take into count when E_g^0 is comparable to energy cutoff of dirac cone.

$$1 = -g_0\int_0^\Lambda \frac{2\pi kdk}{8\pi^2\sqrt{v_0^2k^2+g_0^2\Delta_0^2+\lambda_{so}^2}}$$

$$\frac{4v_0^2\pi}{g_0} = -\sqrt{(v_0\Lambda)^2+g_0^2\Delta_0^2+\lambda_{so}^2}+\sqrt{g_0^2\Delta_0^2+\lambda_{so}^2} \quad (5)$$

$$\frac{4v_0^2\pi}{g_0} = -v_0\kappa+\sqrt{g_0^2\Delta_0^2+\lambda_{so}^2} \quad (6)$$

$$g_0\Delta_0 = \lambda_{so}\sqrt{(\frac{v_0\kappa}{\lambda_{so}}(\frac{4v_0\pi}{g_0\kappa}+1))^2-1} \quad (7)$$

$$\frac{g_0\Delta_0}{\lambda_{so}} = \sqrt{(\alpha(-\frac{g_{0,c}}{g_0}(\frac{1}{\alpha}-1)+1))^2-1} \quad (8)$$

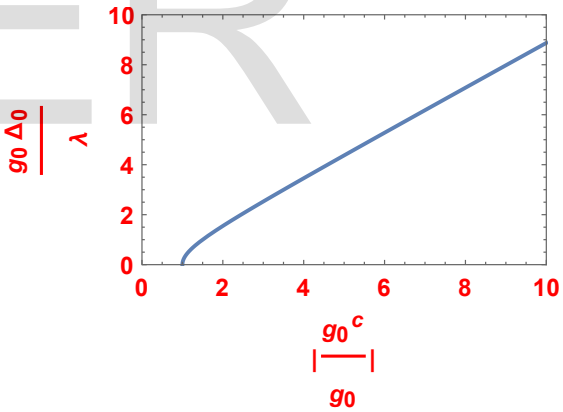

Figure 1: Plot of Eq(7) for $\alpha = \frac{v_0\kappa}{\lambda_{so}} = .1$. It shows that the superconductivity donot exist for $g_0 > g_0^c$. The quantum phase transition predict quantum critical point at finite temperature $(T=0)$ in which the superconductivity does not exist for coupling strength $g_0 > g_0^c$.

Where $g_0^c = -\frac{4v_0^2\pi}{\lambda_{so}-v_0\kappa}$ is critical coupling constant[7][8]. When $g_0 \longrightarrow \infty$, The gap is proportional to $\sqrt{\alpha^2+1}$. The scale invariant equation with re-scaling $x = \frac{g_0\Delta_0}{\lambda_{so}}$

$$F[x,h(g)] = h(g)-\sqrt{1+x^2} = 0 \quad (9)$$

where $h(g) = \alpha(-\frac{g_{0,c}}{g_0}(\frac{1}{\alpha} - 1) + 1)$

$$x = \sqrt{(h(g))^2 - 1} \tag{10}$$

After applying spin index $s = \pm 1$ and converting summation to integration Eq(3) gives

$$\Delta_1(T = 0, g_1, \lambda_{so}) = \frac{1}{12\pi v_0^2 t(1+f^2)^{\frac{7}{2}}}[-\frac{2v_0\kappa f}{3}(v_0^2\kappa^2 - 3)$$

$$+ \frac{2\lambda_{so}^2(f^2-2)\sqrt{f^2-2}}{3} + \lambda_{so}^3 f(1-2f^2)\sqrt{1+f^2}$$

$$- \frac{\lambda_{so}^3(1-2f^2)}{2} ln|\frac{\lambda_{so}f + \lambda_{so}\sqrt{1+f^2}}{-\lambda_{so}f + \lambda_{so}\sqrt{1+f^2}}|$$

$$- \lambda_{so}^2 f(f^2-2)(v_0\kappa - \lambda_{so}\sqrt{1+f^2})$$

$$+ \lambda_{s0}^3 f^2 ln|\frac{\lambda_{so}f + \lambda_{so}\sqrt{1+f^2}}{-\lambda_{so}f + \lambda_{so}\sqrt{1+f^2}}|] \tag{11}$$

Here $f = \frac{g_1\Delta_1}{t}$, for $f^2 << 1$ and making use of leading order approximation, above equation takes form

$$\frac{12\pi v_0^2 t^2}{g_1} = -\frac{2}{3}v_0^3\kappa^3 + 4v_0\kappa\lambda_{so}^2 - \frac{7}{3}\lambda_{so}^3 - \frac{\lambda_{so}^3}{2f} ln|\frac{1+f}{1-f}|,$$

$$\frac{12\pi v_0^2 t^2}{g_1} = -\frac{2}{3}v_0^3\kappa^3 + 4v_0\kappa\lambda_{so}^2 - \frac{7}{3}\lambda_{so}^3 - \frac{\lambda_{so}^3}{2}(1 + \frac{2}{3}f^2),$$

$$f = \sqrt{-\frac{g_1^c}{g_1}(\frac{12v_0\kappa}{\lambda_{so}} - \frac{2v_0^3\kappa_0^3}{\lambda_{so}^3} - \frac{7}{2}) + \frac{12v_0\kappa}{\lambda_{so}} - \frac{2v_0^3\kappa_0^3}{\lambda_{so}^3} - \frac{7}{2}}, \tag{12}$$

$$f = \sqrt{-\frac{g_1^c}{g_1}(12\alpha - 2\alpha^3 - \frac{7}{2}) + 12\alpha - 2\alpha^3 - \frac{7}{2}}. \tag{13}$$

$$f = \sqrt{(12\alpha - 2\alpha^3 - \frac{7}{2})(-\frac{g_1^c}{g_1} + 1)}. \tag{14}$$

Where $g_1^c = \frac{36\pi v_0^2 t^2}{\lambda_{so}^3(\frac{12v_0\kappa}{\lambda_{so}} - \frac{2v_0^3\kappa_0^3}{\lambda_{so}^3} - \frac{7}{2})}$ is the critical coupling constant for p wave.

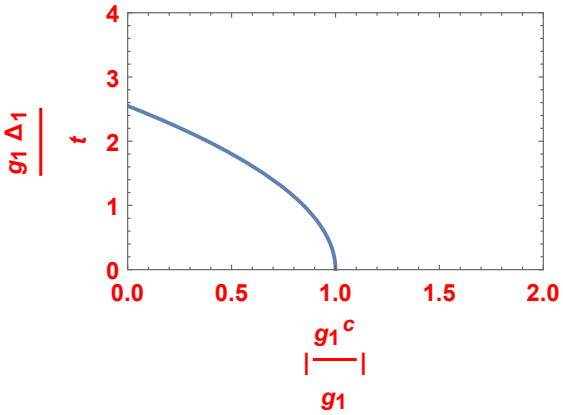

Figure 2: Plot of Eq(13) for $\alpha = 1$. It shows that the superconductivity exist for $g_1 > g_1^c$. The quantum phase transition predict quantum critical point at finite temperature $(T = 0)$ in which the superconductivity exist for coupling strength $g_1 > g_1^c$.

For Transition temperature for s wave consider self consistent equation.

$$1 = -\sum_{k,s}\frac{g_0 \tanh\frac{\omega_{k,s}}{2k_BT}}{4\omega_{k,s}}.$$

After integration we get

$$\frac{4\pi v_0^2}{g} = -K_BT\ln[\frac{\cosh\frac{\sqrt{v_0^2\Lambda^2+\lambda_{so}^2+g_0^2\Delta_0^2}}{2K_BT}}{\cosh\frac{\sqrt{\lambda_{so}^2+g_0^2\Delta_0^2}}{2K_BT}}].$$

At transition temperature $\Delta_0 \longrightarrow 0$,For $x >> 1$:

$$\frac{4\pi v_0^2}{g} = -K_BT_c\ln[\cosh\frac{\alpha}{2K_BT}],$$

$$\frac{4\pi v_0^2}{g} = -K_BT_c\ln[\frac{\exp\frac{\alpha}{2K_BT}}{2}] \tag{15}$$

[1] From Eq. (5), we get

$$\frac{\alpha}{g_0\Delta_0}(-\frac{g_{0,c}}{g_0} + 1) = \sqrt{1 + \frac{1}{x^2}}.$$

For $x >> 1$.

$$\alpha(-\frac{g_{0,c}}{g_0} + 1) = g_0\Delta_0(T = 0).$$

Now Eq. (10) can be written as

$$\alpha(-\frac{g_{0,c}}{g_0} + 1) = K_BT_c\ln 4,$$

$$T_c = \frac{\alpha(-\frac{g_{0,c}}{g_0}+1)}{K_B \ln 4},$$

$$T_c = \frac{g_0 \Delta_0(T=0)}{K_B \ln 4}.$$

$$\frac{2g_0\Delta_0(T=0)}{K_B T} = 2\ln 4 = 2.7725.$$

We note that it does not have any dependence on spin orbit coupling strength. Transition temperature exists for any finite value of coupling strength g_0. The Transition temperature is linearly depend on the zero Temperature gap.

In conclusion, we have derived the $p+ip$ wave gap from electron electron interaction using mean field approximation and find that gap is linearly depend on spin orbit coupling. We find the zero temperature S wave and $p+ip$ superconducting order parameter and find that there exist quantum critical point. Superconductivity exist in s wave in weak coupling regime and for $p+ip$ in strong coupling regime. Weak coupling limit correspond to Fermi liquid behavior in which Fermi surface is large as compare to other energy scales and Fermi sea is unstable to formation of cooper pair mediated by attractive potential, no mater how small is the coupling strength[21]. Strong coupling regime correspond to nodal liquid behavior. We also find the Transition Temperature in Strong coupling regime and observe that Transition Temperature is proportional to zero temperature order parameter. In the limit $g_0 \longrightarrow \infty$, The gap is proportional to $\sqrt{\alpha^2+1}$.

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References

[1] A. A. Abrikosov, L. P. Gorkov, I. E. Dzualoshinski, Method of Quantum field Theory in Statistical Physics, P. 304.

[2] Cheng-Cheng Liu, Hua Jiang, and Yugui Yao PRB **84**, 195430(2011).

[3] Bruno Uchoa and A. H. Castro Neto, PRL **98**, 146801 (2007).

[4] Motohiko Ezawa New Journal of Physics **14**, 033003(2012).

[5] Helene Feldner, itp. uni-frankfurt. de/ jeschke/CMSST/chapter6, Mean field approximation.

[6] Valeri N. Kotov, Bruno Uchoa, Vitor M. Pereira, F. Guinea, and A. H. Castro Neto, Review of modern physics, volume **84**, 1067.

[7] B. Uchoa G. G. Cabrera, and A. H. Castro Neto, PRB **71**, 184509(2005).

[8] A. H. Castro Neto, PRL **86**, 4382(2000).

[9] Motohiko Ezawa, Yukio Tanaka and Naoto Nagaosa Scientific Reports **3**, 2790.

[10] Jason Lee, Wen-Chuan Tian, Wei-Liang Wang and Dao-Xin Yao Scientific Reports **5**, 11512.

[11] Lalmi B, Appl. Phys. Lett. **97**, 223109(2010).

[12] Vogt P., Phys. Rev. Lett **108**, 155501(2012).

[13] Cahangirov S., Phys. Rev.Lett. **102**, 236804(2009).

[14] Guzman-verri G. G. and Lew Yan Voon L. C., Phys. Rev. B **76**, 075131(2007).

[15] Liu C. C., Feng W. and Yao Y., Phys. Rev. Lett. **107**, 076802(2011).

[16] Motohiko Ezawa Phys. Rev. Lett. **109**, 055502(2012).

[17] Jason Alicea Rep. Prog. Phys. **75** 076501(2012).

[18] Wenhui Wan, Yanfeng Ge, Fan yang and Yugui Yao arXiv:1305.3368(2013).

[19] Lan Chen, Baojie Feng and kehui Wu Appl. Phys. Lett. **102**, 081602(2013).

[20] Motohiko Ezawa, Yukio Tanaka and Naoto Nagaosa Scientific Reports **3**, 2790 see supplimentary information.

[21] M. Tinkham, Introduction to Superconductivity, McGraw-Hill, New York, 1996.